เฉลย PAT 1 และคณิตศาสตร์ 1 วิชาสามัญ

สำหรับน้องๆ โครงการพิเศษโรงเรียนราชสีมาวิทยาลัย

ข้อ 18 ตอบ 1

$$5(\sin a+\cos a)+2\sin a\cos a = \frac{1}{25} \rightarrow \sin^2 a+2\sin a\cos a+\cos^2 a+5(\sin a+\cos a) = 1+\frac{1}{25}$$

$$(\sin a+\cos a)^2+5(\sin a+\cos a) = \frac{26}{25} \rightarrow 25(\sin a+\cos a)^2+125(\sin a+\cos a)-26 = 0$$

ให้ $x = \sin a+\cos a$ ดังนั้น $-\sqrt{2} \le x \le \sqrt{2}$

$$25x^2+125x-26 = 0 \rightarrow (5x-1)(5x+26) = 0 \rightarrow x = \frac{1}{5}, -\frac{26}{5}$$

($-\frac{26}{5}$ ใช้ไม่ได้)

ดังนั้น $\sin a+\cos a = \frac{1}{5}$

$$\therefore \quad 125(\sin^3 a+\cos^3 a)+75\sin a\cos a$$

$$= \quad 125(\sin a+\cos a)(\sin^2 a-\sin a\cos a+\cos^2 a)+75\sin a\cos a$$

$$= \quad 125\left(\frac{1}{5}\right)(1-\sin a\cos a)+75\sin a\cos a = 25-25\sin a\cos a+75\sin a\cos a$$

$$= \quad 25(1+2\sin a\cos a) = 25(\sin^2 a+2\sin a\cos a+\cos^2 a)$$

$$= \quad 25(\sin a+\cos a)^2 = 25\left(\frac{1}{5}\right)^2 = 1$$

ข้อ 21 ตอบ 4

$$\tan\frac{\pi}{24} = \tan 7.5^\circ = \frac{2\sin^2 7.5^\circ}{2\sin 7.5^\circ \cos 7.5^\circ} = \frac{1-\cos 15^\circ}{\sin 15^\circ} = \csc 15^\circ - \cot 15^\circ$$

$$= \frac{2\sqrt{2}}{\sqrt{3}-1} - \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \frac{2\sqrt{2}(\sqrt{3}+1)}{2} - \frac{(\sqrt{3}+1)^2}{2} = \sqrt{6}+\sqrt{2}-(2+\sqrt{3})$$

$$= \sqrt{6}+\sqrt{2}-2-\sqrt{3} = (\sqrt{6}-\sqrt{3})-(2-\sqrt{2}) = \sqrt{3}(\sqrt{2}-1)-\sqrt{2}(\sqrt{2}-1)$$

$$= (\sqrt{2}-\sqrt{1})(\sqrt{3}-\sqrt{2}) = (\sqrt{a}-\sqrt{b})(\sqrt{c}-\sqrt{d})$$

$$\therefore \quad a+b+c+d+2 = 2+1+3+2+2 = 10$$

ข้อ 26 <u>ตอบ</u> 2

$$\sin 3A = 3\sin A - 4\sin^3 A \rightarrow \frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$$

$$(3-4\sin^2 9^\circ)(3-4\sin^2 27^\circ)(3-4\sin^2 81^\circ)(3-4\sin^2 243^\circ) = \frac{\sin 27^\circ}{\sin 9^\circ}\times\frac{\sin 81^\circ}{\sin 27^\circ}\times\frac{\sin 243^\circ}{\sin 81^\circ}\times\frac{\sin 729^\circ}{\sin 243^\circ}$$

$$= \frac{\sin 729^\circ}{\sin 9^\circ} = \frac{\sin 9^\circ}{\sin 9^\circ} = 1$$

ข้อ 30 <u>ตอบ</u> 2

$$\sin A + \sin B = 1 \rightarrow 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 1 \quad \text{——}(1)$$

$$\cos A + \cos B = \frac{3}{2} \rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \frac{3}{2} \quad \text{——}(2)$$

$\frac{(1)}{(2)}$ จะได้ $\tan\left(\frac{A+B}{2}\right) = \frac{2}{3}$

$$\therefore \cos(A+B) = \cos 2\left(\frac{A+B}{2}\right) = \frac{1-\tan^2\left(\frac{A+B}{2}\right)}{1+\tan^2\left(\frac{A+B}{2}\right)} = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{5}{13}$$

ข้อ 31 <u>ตอบ</u> 0.5

$$\frac{\cos 36^\circ - \cos 72^\circ}{\sin 36^\circ \tan 18^\circ + \cos 36^\circ} = \frac{\cos 36^\circ - \cos 72^\circ}{2\sin 18^\circ \cos 18^\circ \frac{\sin 18^\circ}{\cos 18^\circ} + 1 - 2\sin^2 18^\circ} = \cos 36^\circ - \cos 72^\circ = \frac{1}{2} = 0.5$$

ข้อ 33 <u>ตอบ</u> 4

$$\arctan\left[\frac{2\cos(60^\circ - 50^\circ) - \cos 50^\circ}{\sin 70^\circ - \sin 10^\circ}\right] = \arctan\left[\frac{2(\cos 60^\circ \cos 50^\circ + \sin 60^\circ \sin 50^\circ) - \cos 50^\circ}{2\cos 40^\circ \sin 30^\circ}\right]$$

$$= \arctan\left[\frac{\sqrt{3}\sin 50^\circ}{\sin 50^\circ}\right] = \arctan\sqrt{3}$$

$$= 60^\circ$$

ข้อ 34 <u>ตอบ</u> 5

$$A = \arctan\left[\frac{2\sin 50^\circ - \cos 20^\circ}{\cos 70^\circ}\right] = \arctan\left[\frac{2\sin(30^\circ + 20^\circ) - \cos 20^\circ}{\cos 70^\circ}\right]$$

$$= \arctan\left[\frac{2(\sin 30^\circ \cos 20^\circ + \cos 30^\circ \sin 20^\circ) - \cos 20^\circ}{\cos 70^\circ}\right] = \arctan\left[\frac{\sqrt{3}\sin 20^\circ}{\sin 20^\circ}\right]$$

$$A = \arctan\sqrt{3} = \frac{\pi}{3}$$

$$\therefore \sin\left(\frac{\pi}{6} + A\right)\cos\left(\frac{\pi}{6} - A\right) = \sin\frac{\pi}{2}\cos\left(-\frac{\pi}{6}\right) = (1)\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

ข้อ 37 <u>ตอบ</u> 1

$$\cot x^\circ = \tan 55^\circ \tan 65^\circ \tan 75^\circ = \frac{\tan 5^\circ \tan 55^\circ \tan 65^\circ \tan 75^\circ}{\tan 5^\circ} = \frac{\tan 15^\circ \tan 75^\circ}{\tan 5^\circ}$$

$$\cot x^\circ = \cot 5^\circ \qquad \therefore \; x = 5$$

ข้อ 38 <u>ตอบ</u> 257

$$\frac{a}{b} = \left(\frac{\cos 6^\circ \cos 54^\circ \cos 66^\circ}{\cos 54^\circ}\right)\left(\frac{\cos 18^\circ \cos 42^\circ \cos 78^\circ}{\cos 18^\circ}\right) = \left(\frac{\frac{1}{4}\cos 18^\circ}{\cos 54^\circ}\right)\left(\frac{\frac{1}{4}\cos 54^\circ}{\cos 18^\circ}\right) = \frac{1}{16}$$

$$\therefore \; a^2 + b^2 = 1 + 256 = 257$$

ข้อ 45 <u>ตอบ</u> 1

$$a_1 = 5,\ a_{50} = 103 \rightarrow a_{50} - a_1 = 98 \rightarrow 49d = 98 \rightarrow d = 2$$

$$\begin{aligned}
a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{49}^2 - a_{50}^2 &= (a_1 - a_2)(a_1 + a_2)(a_3 - a_4)(a_3 + a_4)\ldots\ldots(a_{49} - a_{50})(a_{49} + a_{50}) \\
&= (-d)(a_1 + a_2 + a_3 + a_4 + \ldots\ldots + a_{49} + a_{50}) \\
&= (-2)\frac{50}{2}(a_1 + a_{50}) = -50(5 + 103) \\
&= -5,400
\end{aligned}$$

ข้อ 46 <u>**ตอบ**</u> 500

$$\frac{a_1}{a_1+2} = \frac{a_2}{a_2+3} \rightarrow a_1a_2+3a_1 = a_1a_2+2a_2 \rightarrow a_2 = \frac{3}{2}a_1$$

$$\frac{a_2}{a_2+3} = \frac{a_3}{a_3+4} \rightarrow a_2a_3+4a_2 = a_2a_3+3a_3 \rightarrow a_3 = \frac{4}{3}a_2 = \frac{4}{3}\left(\frac{3}{2}a_1\right) = 2a_1$$

ลำดับชุดนี้คือ $a_1 , \frac{3}{2}a_1 , 2a_1 , ... , a_{1000}$ พบว่าเป็นลำดับเลขคณิต มี $d = \frac{1}{2}a_1$

จาก $a_1+a_2+a_3+......+a_{1000} = 250,000$

$$\frac{1000}{2}(a_1+a_{1000}) = 250,000 \qquad \therefore \; a_1+a_{1000} = 500$$

ข้อ 48 <u>**ตอบ**</u> 1

จากโจทย์ $\frac{S_n}{T_n} = \frac{7n+1}{4n+27}$

จะได้ว่า $S_n = 7n^2+n \rightarrow a_n = 14n-6$ เมื่อ a_n เป็นพจน์ที่ n ของอนุกรมชุดที่ 1

$T_n = 4n^2+27n \rightarrow b_n = 8n+23$ เมื่อ b_n เป็นพจน์ที่ n ของอนุกรมชุดที่ 2

$$\therefore \; \frac{a_{11}}{b_{11}} = \frac{14(11)-6}{8(11)+23} = \frac{148}{111} = \frac{4}{3} \rightarrow a_{11} : b_{11} = 4 : 3$$

ข้อ 49 <u>**ตอบ**</u> 5 : 7

$S_n = 3n^2+4n \rightarrow a_n = 6n+1$

$T_n = 4n^2+9n \rightarrow b_n = 8n+5$

$$\therefore \; \frac{a_9}{b_9} = \frac{6(9)+1}{8(9)+5} = \frac{55}{77} = \frac{5}{7} \rightarrow a_9 : b_9 = 5 : 7$$

ข้อ 50 <u>**ตอบ**</u> 5 : 1

$S_n = 7n^2+2n \rightarrow a_n = 14n-5$

$T_n = n^2+4n \rightarrow b_n = 2n+3$

$$\therefore \; \frac{a_5}{b_5} = \frac{14(5)-5}{2(5)+3} = \frac{65}{13} = \frac{5}{1} \rightarrow a_5 : b_5 = 5 : 1$$

ข้อ 51 **<u>ตอบ</u>** 3.97

$S_n = n^2 + n \rightarrow a_n = 2n$

$T_n = 2n^2 - n \rightarrow b_n = 4n - 3$

$\therefore\ 2\left(\frac{b_{100}}{a_{100}}\right) = 2\left(\frac{4(100)-3}{2(100)}\right) = \frac{397}{100} = 3.97$

ข้อ 52 **<u>ตอบ</u>** 59

$S_n = 3n^2 + 2n \rightarrow a_n = 6n-1 \rightarrow a_{2^n} = 6(2^n)-1$

$$m = \frac{1}{2}a_2 + \frac{1}{2^2}a_{2^2} + \frac{1}{2^3}a_{2^3} + ... + \frac{1}{2^{10}}a_{2^{10}}$$

$$= \sum_{n=1}^{10}\frac{1}{2^n}a_{2^n} = \sum_{n=1}^{10}\frac{1}{2^n}\left[6(2^n)-1\right] = \sum_{n=1}^{10}\left(6 - \frac{1}{2^n}\right)$$

$$= \sum_{n=1}^{10}6 - \sum_{n=1}^{10}\frac{1}{2^n} = 60 - \left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{10}}\right]$$

$$m = 60 - \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^{10}\right]}{1-\frac{1}{2}} = 60 - 1 + \left(\frac{1}{2}\right)^{10} = 59 + \frac{1}{2^{10}} = 59 + \frac{1}{1024}$$

$\therefore$ จำนวนเต็มบวกที่มากที่สุดที่น้อยกว่า $m = 59$

ข้อ 53 **<u>ตอบ</u>** 3

$i^{101} = i$ เพราะ $\frac{101}{4}$ เหลือเศษ 1

$i^{101!} = 1$ เพราะ $\frac{101!}{4}$ ลงตัว

$\therefore\ i^{101} + i^{101!} = i+1 = 1+i$

ข้อ 54 <u>**ตอบ**</u> 1

$$\left(\frac{1+i}{2}-\frac{1}{1+i}\right)^3 = \left(\frac{1+i}{2}-\frac{1}{(1+i)}\frac{(1-i)}{(1-i)}\right)^3 = \left(\frac{1+i}{2}-\left(\frac{1-i}{2}\right)\right)^3 = (i)^3 = -i$$

ข้อ 55 <u>**ตอบ**</u> 2

$$\frac{2+3i}{3-2i} = \frac{(2+3i)}{(3-2i)}\frac{(3+2i)}{(3+2i)} = \frac{13i}{13} = i$$

$$\frac{1+i}{1-i} = \frac{(1+i)}{(1-i)}\frac{(1+i)}{(1+i)} = \frac{2i}{2} = i$$

$$\left(\frac{3i+2}{3-2i}\right)^n = \left(\frac{1+i}{1-i}\right)^{2562} \rightarrow i^n = i^{2562} \rightarrow i^n = i^2 \qquad \therefore\ n = 2$$

ข้อ 56 <u>**ตอบ**</u> 5

(a, b, c) ที่ทำให้ $i^a + i^b + i^c = 1$ แบ่งเป็น 2 กรณี คือ

กรณีที่ 1 (a, b, c) มีค่า a, b, c แตกต่างกัน เช่น $i^1 + i^3 + i^4 = 1$

ซึ่งมีทั้งสิ้น $3! = 6$ แบบ

กรณีที่ 2 (a, b, c) มีค่า a, b, c ซ้ำกัน เช่น $i^2 + i^4 + i^4 = 1$

ซึ่งมีทั้งสิ้น $\frac{3!}{2!} = 3$ แบบ

$\therefore\ n(s) = 6+3 = 9$

ข้อ 57 <u>**ตอบ**</u> 3

$$\frac{1+9i}{5+4i} = \frac{(1+9i)}{(5+4i)}\frac{(5-4i)}{(5-4i)} = \frac{41+41i}{41} = 1+i$$

$$\frac{5-i}{3+2i} = \frac{(5-i)}{(3+2i)}\frac{(3-2i)}{(3-2i)} = \frac{13-13i}{13} = 1-i$$

ให้ $w = \left(\frac{1+9i}{5+4i}\right)^{2019} + \left(\frac{5-i}{3+2i}\right)^{2019} = (1+i)^{2019} + (1-i)^{2019}$

ให้ $z = 1+i$ จะได้ $\bar{z} = 1-i$

จะได้ $w = z^{2019} + (\bar{z})^{2019} = z^{2019} + (\overline{z^{2019}})$

$w = 2\operatorname{Re}(z^{2019})$

จะพบว่า w เป็นจำนวนจริง $\therefore$ ส่วนจินตภาพของ $w = 0$

ข้อ 58 <u>**ตอบ**</u> 120

$|z_1| = 2 \to |z_1|^2 = 4 \to z_1\bar{z}_1 = 4$

$|z_2| = 3 \to |z_2|^2 = 9 \to z_2\bar{z}_2 = 9$

$|z_3| = 4 \to |z_3|^2 = 16 \to z_3\bar{z}_3 = 16$

$$\begin{aligned}|4z_2z_3 + 9z_3z_1 + 16z_1z_2| &= |z_1\bar{z}_1z_2z_3 + z_2\bar{z}_2z_3z_1 + z_3\bar{z}_3z_1z_2| \\ &= |z_1z_2z_3(\bar{z}_1 + \bar{z}_2 + \bar{z}_3)| \\ &= |z_1||z_2||z_3||\bar{z}_1 + \bar{z}_2 + \bar{z}_3| \\ &= (2)(3)(4)|\overline{z_1 + z_2 + z_3}| \\ &= 24|z_1 + z_2 + z_3| = 24(5) \\ &= 120\end{aligned}$$

ข้อ 59 **<u>ตอบ</u>** 0

$|z_1| = 1 \rightarrow |z_1|^2 = 1 \rightarrow z_1\bar{z}_1 = 1$

$|z_2| = 1 \rightarrow |z_2|^2 = 1 \rightarrow z_2\bar{z}_2 = 1$

$|z_3| = 1 \rightarrow |z_3|^2 = 1 \rightarrow z_3\bar{z}_3 = 1$

$(z_1 + z_2 + z_3)^2 = 0 \rightarrow z_1^2 + z_2^2 + z_3^2 + 2z_1z_2 + 2z_2z_3 + 2z_3z_1 = 0$

$z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2z_3\bar{z}_3 + z_1\bar{z}_1z_2z_3 + z_1z_2\bar{z}_2z_3) = 0$

$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) = 0$

$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3(\overline{z_1 + z_2 + z_3}) = 0$

$z_1^2 + z_2^2 + z_3^2 + 2z_1z_2z_3(\bar{0}) = 0$

$\therefore\ z_1^2 + z_2^2 + z_3^2 = 0$

ข้อ 60 **<u>ตอบ</u>** 1

จากโจทย์ $(x-\alpha)(x-\beta)(x-\gamma) = x^3 - x + 1$

แทนค่า $x = -1$ จะได้

$(-1-\alpha)(-1-\beta)(-1-\gamma) = (-1)^3 - (-1) + 1$

$[-(1+\alpha)][-(1+\beta)][-(1+\gamma)] = 1$

$-(1+\alpha)(1+\beta)(1+\gamma) = 1$

$\therefore\ (1+\alpha)(1+\beta)(1+\gamma) = -1$

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